Appendix I: Real Numbers & Inequalities

Example: Solve \( 3x - 4 < 2 \)

highest degree of \( x \) is 1

Linear Inequality

\( 3x - 4 < 2 \)

Transposition & addition

\( 3x < 2 + 4 \)

Solution Set

\( x < 2 \)

Graphical Representation

Set Notation

\( SS = \{ x \mid x < 2 \} \)

Interval Notation

\( x \in (-\infty, 2) \)

Linear Inequality

Example #2: Solve \( 4 - 5x \geq 5 \)

\( 4 - 5x \geq 5 \)

\( -5x \geq 5 - 4 \)

\( -5x \geq 1 \)

\( \{ x \leq -\frac{1}{5} \} \)

Note: Division/Multiplication by a negative (-ve) number requires changing the sense/direction of inequality

Example #3: Solve \( x^2 - 3x \leq 4 \)

highest degree of \( x \) is 2

Quadratic Inequality

\( x^2 - 3x \leq 4 \)

- we need factoring

\( x^2 - 3x - 4 < 0 \)

\( x + 1 = 0 \)

\( (x-4)(x+1) \leq 0 \)

- we need to make one side = 0

\( x = 0 \)

\( (x-4)(x+1) \leq 0 \)

- we need a graphical method

\( SS = \{ x \mid -1 \leq x \leq 4 \} \)

\( x \in (-1, 4] \)
Example #4  \( \frac{x+1}{x+2} \geq 2 \)

**Rational Inequality**

Note: multiplying the expression by \( x+2 \) is not convenient.

Since the sign of \( x+2 \) is not known

\( \frac{x+1 - 2(x+2)}{x+2} \geq 0 \)

\( \frac{x+1 - 2x - 4}{x+2} \geq 0 \)

\( \frac{-x - 3}{x+2} \geq 0 \)

\( \frac{-(x+3)}{x+2} \geq 0 \)

Multiply by -1

\( \frac{x+3}{x+2} \leq 0 \)

**Zero of**

\( x+3 = 0 \) \[
\begin{array}{c|cc}
\text{Zero of} & -1 & -2 \\
\hline
x+3 & - & 0 \\
x+2 & 0 & + \\
\end{array}
\]

\( x+2 = 0 \)

\( x+5 \leq 0 \)

\( x+2 \leq 0 \)

\( x+2 \leq 0 \)

\( x \leq -2 \)

\( \begin{array}{c|c}
\text{Zero of} & -1 \\
\hline
x+3 & -5 \leq x \leq -2 \\
\end{array} \)

\[ x \in [-3, -2] \]

\[ x \in [-3, -2] \]

**Example:** Solve \( \frac{x+1}{x-1} + 1 \geq 0 \)

\( x+1 \quad +1 \geq 0 \)

\( x-1 \)

\( x+1 + (x-1) \)

\( \frac{x+1 + x - 1}{x-1} \)

\( \frac{2x}{x-1} \)

\( x \geq \frac{1}{2} \)

\( x \leq 1 \)

\( x \leq 1 \)

\( x \leq 1 \)

\( x \leq 1 \)

\( x \in [-\infty, 0] \cup x \in (1, +\infty) \)

**Example #5** Solve \( 12(x-4) \leq 9x - 2 \)

\( 2(x-4) \geq 3x - 2 \)

\( \sqrt{2(x-4)^2} \geq \sqrt{(3x - 2)^2} \)

\( 2(x-4)^2 \geq (3x - 2)^2 \)

\( 4(x^2 - 8x + 16) \geq 9x^2 - 12x + 4 \)

\( 4x^2 - 92x + 64 \geq 9x^2 - 12x + 4 \)

\( -5x^2 - 20x + 60 \geq 0 \)

\( x^2 + 4x - 12 \leq 0 \)

\( (x+6)(x-2) \leq 0 \)

\( x^2 + 20x - 60 \geq 0 \)

\( x^2 + 4x - 12 \leq 0 \)

\( (x+6)(x-2) \leq 0 \)

\( x \geq -2 \)

\( x \leq -6 \)

\( x \geq -6 \)

\( x \leq -2 \)

\( x \in (-\infty, -6] \cup x \in [-2, +\infty) \)
Example I show that the points $A(3, -6)$, $B(5, 2)$, and $C(-1, 1)$ form a right triangle. Find the length of the median from $A$ to $BC$.

**Pythagorean Theorem**

| $|AB|^2$ | $|AB|^2 + |AC|^2$ | $|BC|^2$ |
|---------|----------------|---------|
| $(3-5)^2 + (-6+2)^2$ | $(3-5)^2 + (-6+2)^2$ | $(5+1)^2 + (-2+1)^2$ |
| $82$ | $82$ | $82$ |

**Distance Formula**

| $PQ$ | $|PA|^2 + |AC|^2$ |
|------|----------------|
| $PQ = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$ | $|PA|^2 + |AC|^2$ |

**Midpoint Formula between $P$ and $Q$**

| $M(x, y)$ | $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ |

**Verify Pythagorean theorem**

| $180^\circ$ | $|AB|^2 + |AC|^2$ |
|------------|----------------|
| $180^\circ$ | $|AB|^2 + |AC|^2$ |

$82^2 + 82^2 = 82^2 + 82^2$; $ABC$ is a right triangle because it satisfies the Pythagorean theorem. $ABC$ is an isosceles $\triangle$.

**Midpoint between $B$ and $C$**

| $M(x, y)$ | $\left(\frac{3-1}{2}, \frac{-2-1}{2}\right)$ |

**Distance Formula**

| $|AM|$ | $\sqrt{(x-3)^2 + (y+6)^2}$ |
|-------|----------------|
| $|AM|$ | $\sqrt{(x-3)^2 + (y+6)^2}$ |

Note:

- Type 1: $ax + by + c = 0$
- Type 2: $y = ax^2 + bx + c$
- Type 3: $x = ay^2 + by + c$
- Type 4: $ax^2 + by^2 + cx + dy + e = 0$ ($a = b$)

**Graphs of Equations**

- Linear Equation in $x$ and $y$ line
- Linear in $y$, quadratic in $x$ parabola
- Quadratic in $x$, quadratic in $y$ circle/point/empty set ($a = b$)

**Given:** $3x + 2y - 12 = 0$

- a. sketch its graph
- b. identify its graph (give it a name)
- c. find its slope and its $x$ and $y$ intercepts
November 20, 2001 (Tuesday)

Appendix #2:

@ Type 2 @ Parabola

Ex. \( y = x^2 + 2x + 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

\( \sqrt{\text{ substitute}} \)

\( \text{Domain: } x \in \mathbb{R} \)

\( \text{Range: } y \geq 0 \quad y \in [0, +\infty) \)

8. points in a parabola has a mirror image on the opposite side of the axis of symmetry

9. vertex - unique point lying on an axis; the mirror image of itself

10. axis of symmetry is identified with the zero of the "perfect square" - any quantity raised to even power is a perfect square

\( x = k \) is a vertical line

\( \downarrow \) magic number

\( \text{magic number } = \left( \frac{1}{2} \right) \text{ of the coefficient of } x^2 \)

\( \sqrt{\text{ substitute in the equation}} \)

\( \text{Note: } y = ax^2 + bx + c \)

| \( a > 0 \) | concave up | parabola |
| \( a < 0 \) | concave down | parabola |

Ex. \( 3 \quad x = ay^2 + by + c \) parabola

| \( x \) | -1 | 0  |
| \( y \) | 5  | 5  |

Note: for domain \( y \geq 0 \)

\( x \geq 2 \quad 3 + \text{dom.} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Ex. 2 \( y = 3 - 4x - 4x^2 \) parabola

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Ex. \( 3 \quad x = ay^2 + by + c \) parabola

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\( y = -4 \left( x + \frac{1}{2} \right)^2 + 6 \) zero of the perfect square

\( \text{vertex: } \left( -\frac{1}{2}, 6 \right) \)

| \( \text{Domain: } x \in \mathbb{R} \) |
|---|---|

| \( \text{Range: } y \leq 6 \) |
|---|---|

| \( y \in (-\infty, 6] \) |
The image contains a mathematical lecture on the topic of circles and straight lines. It includes the general equation of a circle, how to complete the squares, and the process of finding the center and radius of a circle. The document also covers the concept of parallel lines and how their slopes are related. Additionally, it discusses how to check if a point lies on a circle and introduces the concept of perpendicular lines. The document is written in English and is legible, with clear diagrams to aid understanding.
1. Consider the circle \( x^2 + y^2 + 6y - 12 = 0 \)
   a. Sketch its graph
   b. Find the equation of the line passing through the point \( P(5, 1) \) and its center.
   c. Find the equation of the diameter parallel to the line \( 2y - 4x - 3 = 0 \)

2. Sketch the graph of \( y = (x+1)^2 + 1 \) after analyzing for symmetry, domain, and range.

3. Solve the inequality \( \frac{x+1}{x-1} + 1 \geq 0 \)

4. Complete the squares
   \( (x^2 - 4x + 4) - 4 \)
   \( (x-2)^2 - 4 + (y+3)^2 - 9 = 12 \)
   \( (x-2)^2 + (y+3)^2 = 25 \)

Center: \((2, -3)\) Radius: 5 units

5. Solve for the points
   a. \( y = mx + b \)
   b. \( 2y - 4x - 3 = 0 \)
   c. \( y = 2x + 3 \)
   d. \( 2x - y - 7 = 0 \)

6. Find the zero of \( x = -1 \)

7. \( x = (x+1)^2 + 1 \)

8. Substitute 
   \( y = 1 \)

9. Concave upward

10. \( v(-1, 1) \)

11. \( y = 2x + \frac{3}{2} \)

Slope of Line
\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]
\[
+ \quad - \quad -
\]
\[
\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}
\]
\[
m = \frac{1}{3}
\]

Checking:
Point-Slope Formula
\[
y - y_1 = m(x - x_1)
\]
\[
y - y_1 = m(x - x_1)
\]
\[
-4 = - \frac{4}{3} \quad \frac{3}{3} y = 4x + b
\]
\[
-4 = - \frac{4}{3} \quad \frac{3}{3} y = 4x + b
\]
\[
4x - 3y + 6 = 0
\]

Least Common Denominator
\[
\frac{X+1+x-1}{x-1} \geq 0
\]
\[
\frac{X-1}{x-1} - \frac{X}{x-1}
\]
\[
X \rightarrow 0 \quad x \in (-\infty, 0] \cup [1, +\infty)
\]